DESIGN OF AN INSTRUMENT TO MEASURE THE SHEAR MODULUS OF SOFT SEDIMENTS

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THESIS

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by

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ABSTRACT

The design considerations and the construction details of a torsionally oscillating vaned probe viscoelastometer to be used for the measurement of the shear modulus of soft sediments are presented. Its operation involves measuring the reaction to the radiation of shear waves into the sediment from a vaned cylindrical probe which executes simple harmonic torsional oscillations. The radiation reaction should be sensitive to the viscoelastic parameters of the sediment. Torque and angular velocity sensors at the probe head provide a means of measuring the mechanical impedance. Sensor calibration procedures and their results are described. The instrument operates at frequencies in the neighborhood of 900 and 2700 Hz.

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ACKNOWLEDGEMENTS

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I. INTRODUCTION

A model of the ocean floor which can accurately predict transmission and reflection properties is essential if contemporary sonar techniques, such as bottom bounce mode, are to be exploited to the maximum extent possible. To formulate such a model requires not only a detailed knowledge of the viscoelastic properties of ocean floor soils, but also that this knowledge be acquired through in situ measurements, rather than through laboratory measurements, so that it is truly representative of the ocean environment.

If the sediment is capable of supporting a shear wave, then a technique which provides for the generation of such a wave and then provides a means of measuring the complex Lamé constants of the medium would provide not only the information which is sought, but should also be easily adaptable to in situ measurements. Such a technique was first utilized by Mason [6] to measure the viscosity and shear elasticity of various liquids. The torsional wave viscoelastometer method described by Mason was modified by Cohen [2] and Hutchins [3] and its use extended to the measurement of the complex shear modulus of soft sediments. Hutchins investigated the operation of the viscoelastometer in the resonance mode proposed by Mason and in a pulsed mode proposed by McSkimin [8]. Both techniques yielded a complex shear modulus for the laboratory water-saturated

sediment investigated (kaolinite), thus indicating that the sediment exhibits both viscous and elastic properties, and further that the viscoelastometer is a useful technique for measuring these properties.

Utilizing an improved design for the viscoelastometer, Bieda [1] extended the use to several samples of continental terrace clayey-silt sediments. His measurements were performed in the laboratory along with measurements of other mass-physical properties of the samples. A correlation of his data indicated that the real part of the complex modulus was consistent with the results obtained by other researchers utilizing different techniques.

Additional study of the viscoelastometer at the Naval Postgraduate School by Lasswell [5] has tended to confirm the validity of the technique, but has also revealed a severe temperature dependence in the existing instrument which renders it virtually impractical for in situ measurements.

It has been the intent of the research described in this report to develop and construct a viscoelastometer less sensitive to temperature and thus more suitable for in situ measurements than the instruments previously used. One feature of the existing method is retained: measurement of the reaction on the torsionally vibrating probe due to the radiation of shear waves into the medium in which the probe is imbedded. Torque and angular velocity sensors located

at the immersed head of the probe provide data from which the specific radiation impedance for shear wave in the sediment can be computed.

II. THEORY

The viscoelastometer is based on the principle that a cylindrical probe executing simple harmonic torsional motion about its axis will generate shear waves in the sediment in which it is immersed and that the reaction of the sediment upon the probe due to the radiation of these waves can be measured and related to the viscoelastic properties of the sediment.

In developing and confirming this technique initially, Mason [6] utilized a cylindrical transducer supported at its middle (nodal) plane. In the investigations of Cohen [2] and Hutchins [3] the device was modified to include a metallic, cylindrical rod attached to the torsional drive transducer. The assembly was suspended by a thin fiber attached to the upper (transducer) end. Bieda [1] and Lasswell [5] returned to the centerplane design but with symmetric, metallic probes attached to either end of the drive transducer which was suspended by a resilient clamp at its center. While the theory of operation is unaffected by the method of suspension or location of the transducer, center plane suspension does facilitate establishing an exact nodal plane and reduces energy losses to the support. End point support makes possible in situ measurements with the least disturbance of the sediment. Location of the transducer at the center also improves its electromechanical coupling.

Following the method developed by Mason the mechanical impedance of a sediment is obtained by assuming pure torsional motion of the rod, no slippage at the rod-sediment interface, and that the wavelength of the generated shear wave is much smaller than the radius of the rod. The generated shear wave can then be considered to be plane of the form:

$$\tau = \tau_o \exp[\sqrt{\pi f \rho / \eta}\ (1 + i)r], \tag{1}$$

where τ is the shear stress amplitude, τ_o is the initial amplitude, f is torsional oscillatory frequency, ρ is the density of the immersed medium, η is the coefficient of shear viscosity, and r is distance along the direction of propagation.

In the case of a Newtonian fluid the coefficient of shear viscosity is given by: $\eta = \tau/\dot{S}$ where S is the shear strain and $\dot{S} = \partial S/\partial t$. If the fluid is assumed to be viscoelastic then η is complex and:

$$\eta_c = \eta_1 - i\eta_2. \tag{2}$$

This same viscosity can also be described in terms of a rigidity modulus, G, which, for the viscoelastic medium, is also complex:

$$G_c = G_1 + iG_2. \tag{3}$$

Because the motion of the rod is simple harmonic, S is of the form: $S = S_o e^{i\omega t}$ and hence, $\dot{S} = i\omega S$. Therefore:

$$\eta_c = \frac{\tau}{i\omega S} = -\frac{iG_c}{\omega} \tag{4}$$

The specific radiation impedance of the sediment (z) is defined as the ratio of shear stress to particle velocity and is given by:

$$z = \sqrt{\pi f \eta \rho}\,(1 + i) = R + iX \tag{5}$$

where R and X are the specific shear wave resistance and reactance of the sediment respectively. By substituting the complex shear viscosity coefficient and separating real and imaginary parts, Mason [6] has shown that the following relationships exist:

$$\begin{aligned} \eta_1 &= \frac{2RX}{\omega \rho_{sed}} & \qquad G_1 &= \frac{R^2 - X^2}{\rho_{sed}} \\ \eta_2 &= \frac{R^2 - X^2}{\omega \rho_{sed}} & \qquad G_2 &= \frac{2RX}{\rho_{sed}} \end{aligned} \tag{6}$$

and the problem of determining the viscosity and shear modulus reduces to measuring R and X.

Mason [6], Cohen [2], Hutchins [3] and others have shown that this can be accomplished by relating R and X to

changes in the resonance frequency and mechanical Q of the torsional rod. In the instrument described herein, the mechanical impedance Z of the cylinder imbedded in the sediment is to be measured directly utilizing torque and angular velocity sensors.

If it is assumed that at the rod-sediment interface the sedimentary particles maintain contact with the rod surface with no slippage then the shear stress is related to the torque (T) applied to the rod segment in contact with the sediment and the shear strain is related to angular velocity ($\dot{\theta}$) of the rod, then:

$$Z \propto \frac{|T|}{|\dot{\theta}|} e^{i\phi} \qquad (7)$$

where ϕ is the phase angle between T and $\dot{\theta}$.

If the assumptions used in deriving equation (1) are valid, that is, if the shear waves radiating into the medium surrounding the cylinder are rapidly damped and are essentially plane, then the torsional mechanical impedance (Z) due to radiation of the shear waves should be given by:

$$Z = (R + iX)\, Aa^2 \qquad (8)$$

where A is the surface area of the cylinder exposed to the medium and a is its effective radius. Thus a combination of equations (5), (6), (7), and (8) provide a means for

evaluating the real and imaginary components of the shear modulus in terms of the observed mechanical impedance.

The proposed method requires experimentally determined calibration curves for the angular velocity and torque sensors of the viscoelastometer. An optical measurement of the mechanical displacement of the rod, at a known frequency, as a function of velocity sensor voltage provides the calibration for the velocity sensor. The torque signal can then be calibrated by attaching masses of known moments of inertia to the head of the probe and obtaining data so that the calibration constant can be solved for analytically. Once calibrated the problem of determining Z is then reduced to measuring the torque, angular velocity and the phase difference between them.

III. THE VISCOELASTOMETER

A. GENERAL DESCRIPTION

Figure 1 is a sketch of the viscoelastometer. It consists of: (1) a metallic head 3 inches long, with vanes, which will execute torsional oscillations and generate shear waves in the sediment and is tapered to minimize disturbance of the sediment upon insertion; (2) an electro-dynamic velocity sensor enclosed within the oscillating head; (3) a torque sensor consisting of a 2-inch long barium titanate torsional transducer designed to produce an electrical signal which is proportional to torsional stress; (4) a stainless steel tube 26 inches long and 0.75 inches O.D. (both ends of the rod were "plugged" and the lower end bored for the passage of wiring from the torque and velocity pick-up); (5) a hollow cylindrical barium titanate transducer 3 inches long fabricated to produce torsional motion when driven electrically; (6) a massive supporting flange with handles for insertion of the device into the sediment and a "screw-type" fitting for the addition of either a mass or a resonant system to provide a reaction for the drive transducer which will tend to cause the upper end of the drive transducer to be a nodal plane so that the length from that point to the head is nearly one-quarter of a wave-length long at the first resonant mode.

B. THE DRIVE HEAD

Figure 2 is a sketch of the drive head of the probe. It consists of a steel head 3 inches long, 0.75 inches O.D. and tapered at one end for insertion into the sediment with a minimum of disturbance. Four tapered vanes 1.5 inches long, 0.2 inches wide and 0.03 inches thick are located in quadrature. The head has a bore of 0.5 inches in diameter and 1.25 inches deep for placement of the velocity pick-up. The head was constructed in two pieces with a screw-thread fitting and "o-ring" seal to permit access to the velocity pick-up and to permit changing of the vane-section to heads of various vane size.

In designing the head the selection of an optimal length was constrained by the fact that the length of the head and the barium titanate torque pick-up should be small compared to the resonant wavelength of the torsional standing-wave so that the variations in torque and angular velocity along the "active" region are negligible. Coupled with this constraint is the additional restriction that the head be large enough to enclose a velocity sensor of practical dimensions. The tapered vanes were included to give the head the properties of the vane shear device utilized in engineering properties studies. Since the head is removeable it is envisioned that later research will investigate the effects of various vane sizes and shapes.

C. THE VELOCITY PICK-UP

A sketch of the velocity pick-up is shown in Figure 3. It consists of a ferromagnetic frame inside of which is wound a coil consisting of 168 turns of 36 gauge wire with a measured resistance of 11 ohms. A small iron-cobalt permanent magnet is suspended at the center of the frame with a "soft" fiber suspension so that the torsional motion of the rod does not couple to the magnet and it acts as essentially an inertial element at frequencies of interest. The magnet is oriented within the frame for maximum flux linkage. Side pieces of lucite are attached to the frame so that the device is cylindrically symmetric with a 0.5 inch O.D. and is 0.75 inches long. The pick-up is then inserted into the drive head and secured with a set screw.

D. THE BARIUM TITANATE CERAMICS

Both the drive transducer and the torque sensor are constructed from commercially available ceramics (see Gulton Industries Inc. or Vernitron Piezoelectric Division catalogs) which are 0.75 inch O.D. and 0.5 inch I.D. Mason [7] has shown that to achieve torsional vibration the axially polarized ceramics may be sliced longitudinally and then reassembled to achieve the residual polarization shown in Figure 4. The joints were cemented with epoxy resin and each joint contains a thin wire mesh to control glue joint thickness and to establish the excitation field. The drive transducer was constructed with external tabs, as shown in Figure 4, for the electrical connections. The torque sensor was

constructed with internal tabs. After assembly of the probe the external surfaces of both ceramics were coated with an acrylic lacquer for electrical insulation and waterproofing.

E. MECHANICAL ASSEMBLY

In assembling the probe all joints were "butt" fitted and cemented utilizing Techkits A-12 eposy resin. A nylon mesh gasket approximately 5 mils thick was utilized at each joint to maintain a uniform glue joint thickness. A threaded rod attached to one end of the 26-inch rod and which passes through the drive transducer and into the supporting flange provides additional mechanical strength at this large stress region and also provides a means of applying a controlled static compression to the drive transducer. All wiring for the torque and velocity sensors is internal to the device with access through two small holes at the top of the stainless rod.

To minimize stray electrical pick-up all external wiring was shielded with metallic braid. A common electrical ground was provided by utilizing conducting epoxy strips across the insulated ceramics to electrically connect all external metallic components.

To insure the water-tight integrity of the lower assembly the velocity sensor set-screw access was sealed. Duco cement was utilized because it provides the necessary seal and is, at the same time, soluble in acetone and other solvents.

F. ADDITIONAL DESIGN CONSIDERATIONS

As previously mentioned the lengths of the ceramics and the metallic head are fixed by either their commercial availability or by specific design constraints. Thus in determining what length the device should be for a given resonant frequency the only variable is the length of the stainless rod.

In designing the device the ceramics were selected to insure sufficient torsional drive at relatively low applied voltages and sufficient response from the torque sensor. The metallic head was then designed within the constraints previously mentioned. Because a first resonant frequency of approximately 1000 Hz was desired, the length of the stainless rod was obtained by the following procedure:

1. The probe was initially considered to be a homogeneous rod fixed at one end and free at the other. Thomson [9] has shown that for a fixed frequency, the length is given by:

$$L = \frac{\pi}{2\omega} \sqrt{\frac{G}{\rho}} , \qquad (9)$$

where G is the rigidity, ρ is the density of the rod and ω is the angular frequency. Utilizing values of G and ρ for type 347 stainless, L was found to be 82.5 cm.

2. The probe was then analyzed as a cylindrical rod 77.6 inches long which is fixed at one end and which has a solid cylinder 4.9 cm long attached at the other. Following the treatment given in Thomson [9] the resonant frequency

is found by considering

$$\beta \tan \beta = \frac{J_{rod}}{J_o} , \tag{10}$$

where J_{rod} is the moment of inertia of the rod, J_o is the moment of inertia of the cylindrical mass and

$$\beta = \omega L \sqrt{\frac{\rho}{G}} . \tag{11}$$

Then $$f = \frac{\beta}{2\pi L} \sqrt{\frac{G}{\rho}} . \tag{12}$$

Again utilizing the appropriate values for type 347 stainless f is found to be 925 Hz.

3. Since the rod is not homogeneous and J_o is not known exactly the error in the above analysis is found by considering equation (12):

$$\frac{df}{f} = \frac{d\beta}{\beta} + \frac{dG}{G} \tag{13}$$

Since G (barium titanate) ≈ G (347 stainless), $\frac{dG}{G}$ is approximately zero and $\frac{df}{f} \approx \frac{d\beta}{\beta}$. From equation (10) and assuming the worst possible case that J_o is only known to within a factor of 2, it was found that $\frac{d\beta}{\beta} = 0.038$ and $f = 925 \pm 35$ Hz.

G. LIMITATIONS OF THE DEVICE

One of the assumptions inherent in the design of the viscoelastometer is that the region over which torque and angular measurements are made is small compared to the

wavelength of the torsional standing wave along the rod, and hence the angular velocity is essentially uniform. Since this region is 3 inches long and at the first resonant mode of 925 Hz the wavelength of the torsional wave is approximately 120 inches, this is a valid assumption. At the third resonance the assumption remains valid but beyond that the approximation no longer holds. Thus the useful range of the probe is approximately 900 - 2700 Hz. To extend that range would involve miniaturizing of the torque and velocity pick-ups even more than they are now.

While the intent of the research described in this report has been to develop a technique for measuring the viscoelastic properties of water-saturated sediments in situ the viscoelastometer described is extremely fragile for in situ measurements. It is, rather, a means of verifying the technique for measuring the radiation impedance; "hardening" of the device can come later. Specific areas of weakness in the probe are; (1) mechanical weakness of all cemented joints, and (2) fragility of the velocity pick-up.

IV. TEST RESULTS AND CALIBRATION TECHNIQUES

The diagram shown in Figure 5 represents the experimental set-up utilized in testing and calibration of the probe. Several features of this set-up are critical and worthy of further explanation. The type and length of the leads from the torque and velocity sensor are shown because it was found that these leads are critical to the calibration and operation of the device because of phase shifts introduced by the cables. The torque signal is first passed through a vacuum tube voltmeter (VTVM) used as a voltage amplifier. Not only is this a convenient method of measuring the amplitude of the signal but more importantly the amplifier serves as an impedance matching device thus minimizing the electrical loading on the torque sensor. In spite of the precautions taken to minimize stray pick-up, a considerable amount of noise was present, particularly on the velocity signal. Hence, pass band filters are utilized to filter out both high and low frequency noise.

Quite naturally the first test conducted on the device was a check of resonance. The primary resonance of the probe occurs at 906 Hz and thus is within the uncertainty limits of the calculated resonance. Third harmonic resonance occurs at 2720 Hz. Thus the useful range of the device is approximately 900 - 2700 Hz. Actually, useful output from the sensors is obtained only in the neighborhood of the resonance frequencies.

A qualitative check of probe performance was next conducted. At a fixed level of drive voltage the torque and velocity signals were observed on a dual-trace oscilloscope while the device was in air. The head of the probe was then immersed in mud and at the same drive level the signals were again observed on the scope. Figure 6 shows samples of the observed signals. Although these photographs are not quantitatively significant they do clearly illustrate the reduction in signal strength and a measurable phase shift when the device is immersed in mud. Also observed was the measurable shift in resonant frequency so instrumental to the work of Mason [6], Cohen [2], Bieda [1], and others.

Having confirmed that the device was operable it was then necessary to determine if the device could, in fact, be calibrated. In determining whether or not the probe could be calibrated it was reasoned that if, at resonance, the torque and velocity signals were plotted against drive voltage and a linear relationship was found to exist over all or part of the range, then the two signals could be calibrated. Two data sets were obtained and are plotted in Figures 7 and 8. Clearly the mandatory linear relationships exist and thus the device can be calibrated. The difference in the slope between data sets I and II is believed to be due to data set I being taken when the drive was slightly off resonance. The cause of the discontinuity in curve II which occurs between 14 and 15 volts drive is unknown but the fact that it occurs in both the torque and velocity

signals would seem to indicate that its source lies in the upper portion of the probe. A possible explanation might be that it represents a mechanical slippage at one of the threaded joints.

Prior to proceeding with a calibration of the device it was considered imperative that the torque and velocity signals of the probe immersed in mud be plotted as a function of drive voltage. Again one would expect a linear relationship to exist between the signals and the drive voltage, linear at least to the point where the motion of the rod becomes sufficiently large that slippage at the rod-sediment interface occurs. As Figures 9 and 10 indicate, this linear relationship is clearly evident throughout the range of drive voltages considered. Thus, not only can the device be calibrated but it can also be used in mud at any drive level up to and including 20 volts without violating the approximations inherent in the theory of operation.

The following procedure was utilized to calibrate the angular velocity sensor. Utilizing a Unitron type MeC3 -2332 microscope with internal illumination and calibrated grid, the actual peak-to-peak physical displacement of the probe head was observed as a function of velocity sensor output. The displacement data were then converted into peak angular velocities ($\dot{\theta}$) and plotted as a function of sensor output voltage (in RMS volts). Figure 11 presents the observed data in graphical form. Utilizing the 20 observed data points and the origin, least square fitting of a regression

line was performed. The slope of this line then represents the velocity sensor calibration constant. Experimental uncertainty was then determined by statistical methods. As a result of this procedure the calibration constant was found to be $K_\theta = 7.46 \pm 0.092$ mv^{-1}-sec.$^{-1}$ (1.2% accuracy).

The procedure for calibrating the torque sensor involves adding disks of known moment of inertia to the oscillating head and observing the torque and velocity signals. Since the mechanical system is completely analogous to a series inductive circuit if dissipative losses can be ignored, the calibration constant can be determined analytically. Appendix I tabulates the observed data, presents a derivation of the analytical solution, and a sample calculation. Since the number of data points was insufficient for a statistical determination of precision the derived equation was utilized to determine differential errors. As a result of this procedure the calibration constant was found to be $K_T = 7.12 \pm 0.51\text{x}10^5$ dyne-cm-volt^{-1} (RMS) (7.2% error).

V. RECOMMENDATIONS

It is imperative to remember that the viscoelastometer described in this paper is calibrated for use with a specific equipment hook-up. Any changes to that hook-up will require a complete recalibration. The device is thus severely limited but, because of the simplicity of the calibration procedure, this is not considered a critical deficiency.

Although an approximate relationship between the measured mechanical impedance and these physical properties has been described in Chapter II, the theory has not been fully developed for this particular device. It is recommended that a more precise relationship be developed.

Because of the fragility of the present device it should be redesigned to strengthen it for in situ measurements. This would involve strengthening of all mechanical joints and improvement of the velocity pick-up. The latter might be accomplished by utilizing another torsional ceramic mounted so as to serve as an inertial element. While this would measure angular acceleration, the simple harmonic nature of the driving function makes this easily convertible to angular velocity.

The present probe has a useful range of 900 - 2700 Hz. Consideration should be given to extending the range of the device. While this would involve miniaturizing of the velocity sensor this is not considered an insurmountable problem particularly if a torsional ceramic is utilized as the sensor.

Throughout this research instrumentation has presented severe problems. The small signals produced by the velocity pick-up makes induced noise a particular problem, especially when attempting to accurately measure phase shifts. The output voltage and phase of the torque sensor are very sensitive to the electrical loading of the connecting cables and measuring instruments. Tentative solutions were found but they all involved the addition of bulky, costly equipment. Present day integrated circuit technology makes many of the problems experienced correctable by less costly, more efficient means. Signal boosting and impedance matching devices can quite easily be incorporated within the space limitations of the present probe, and it is recommended that considerable effort be directed in this area.

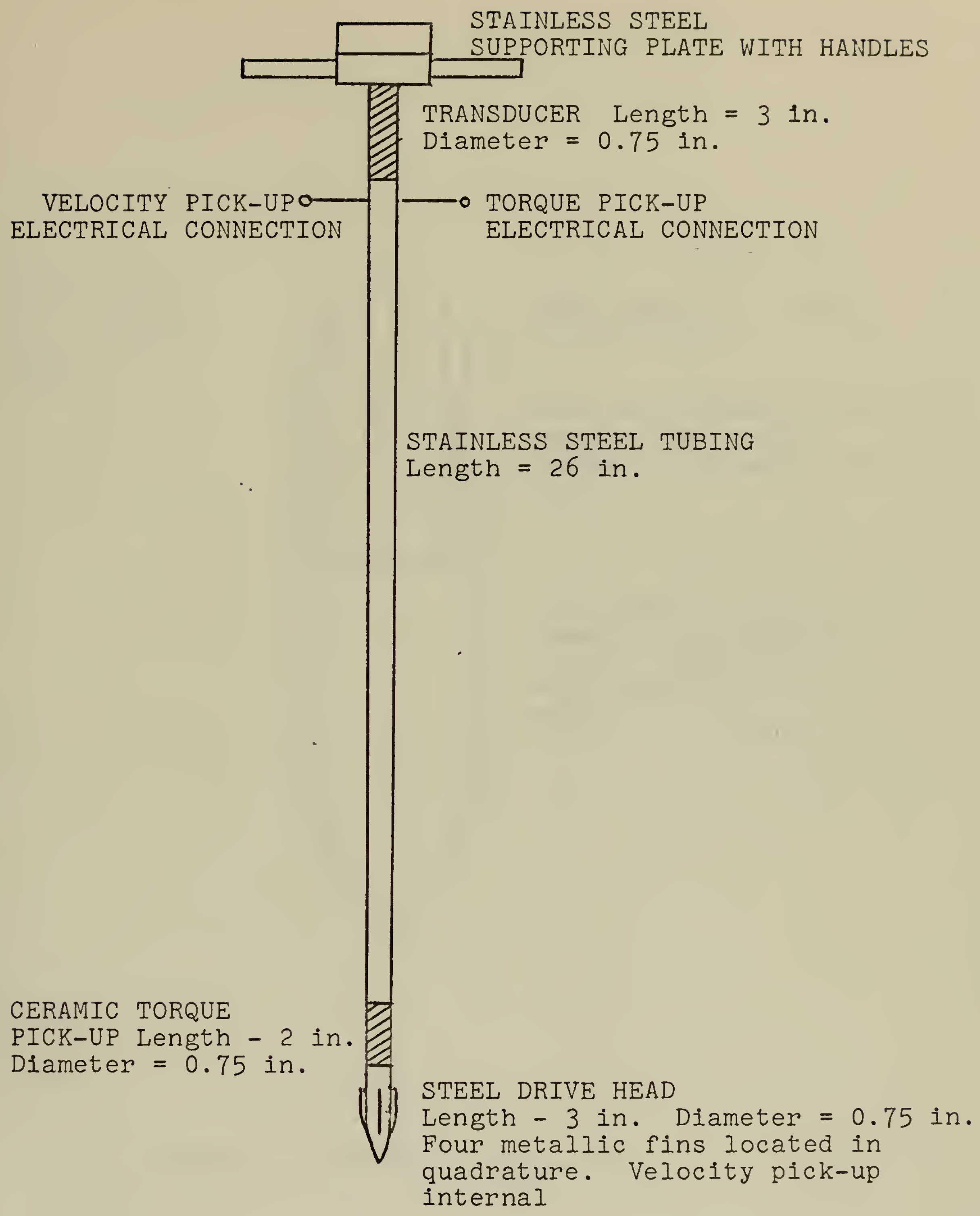


Figure 1. Sketch of Viscoelastometer

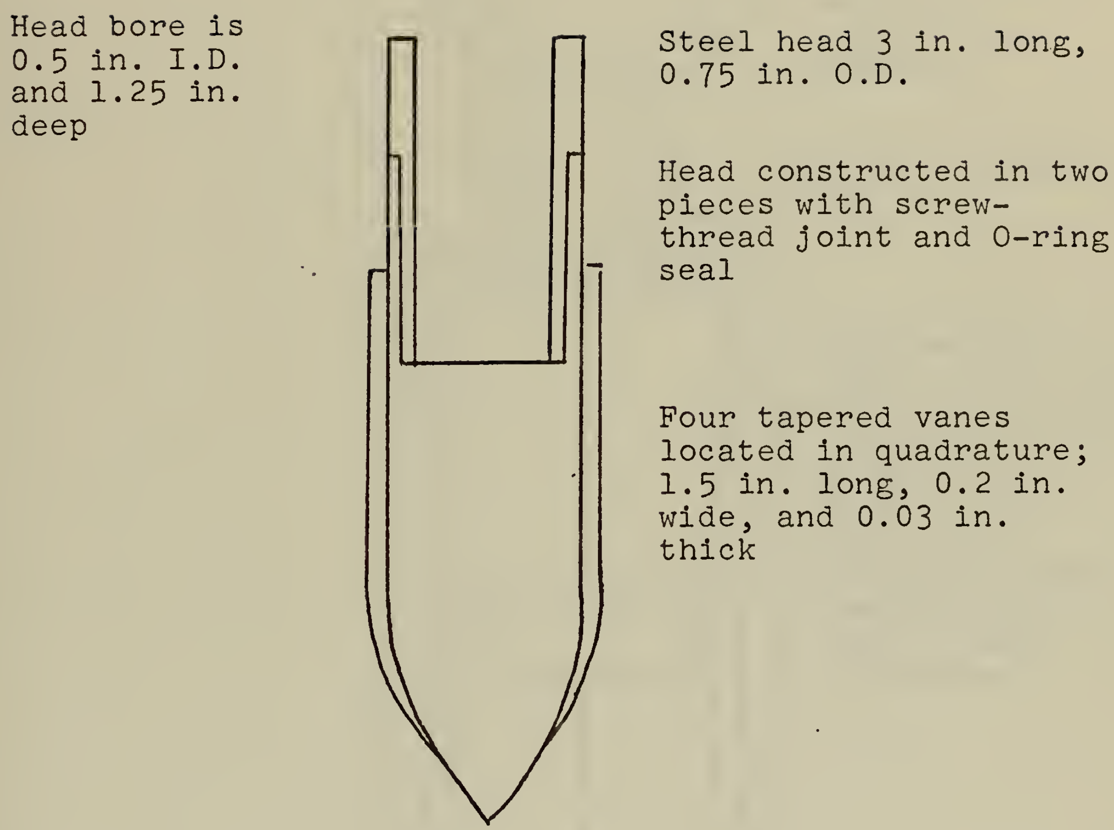


Figure 2. Sketch of Drive Head

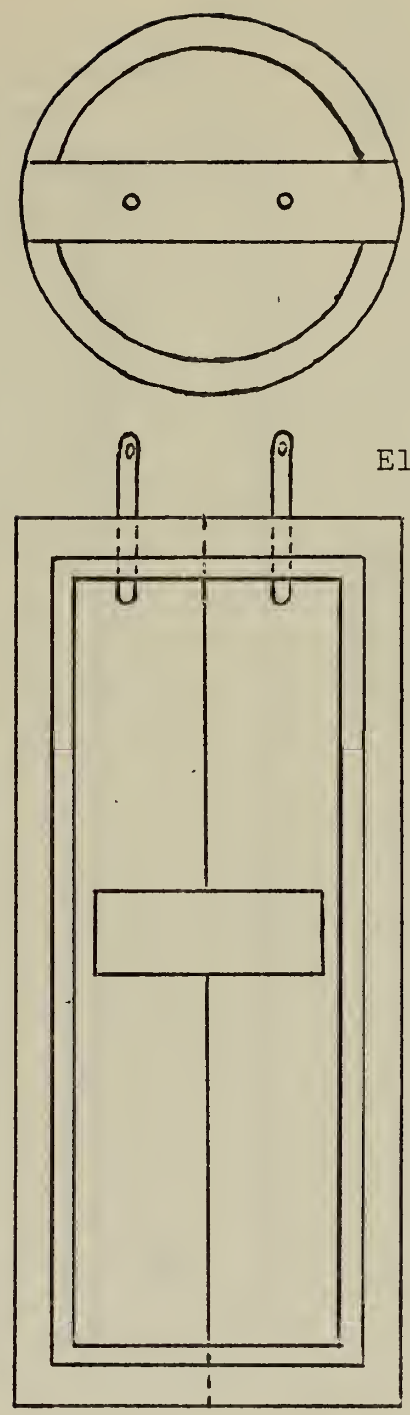


Figure 3. Sketch of Velocity Pick-Up

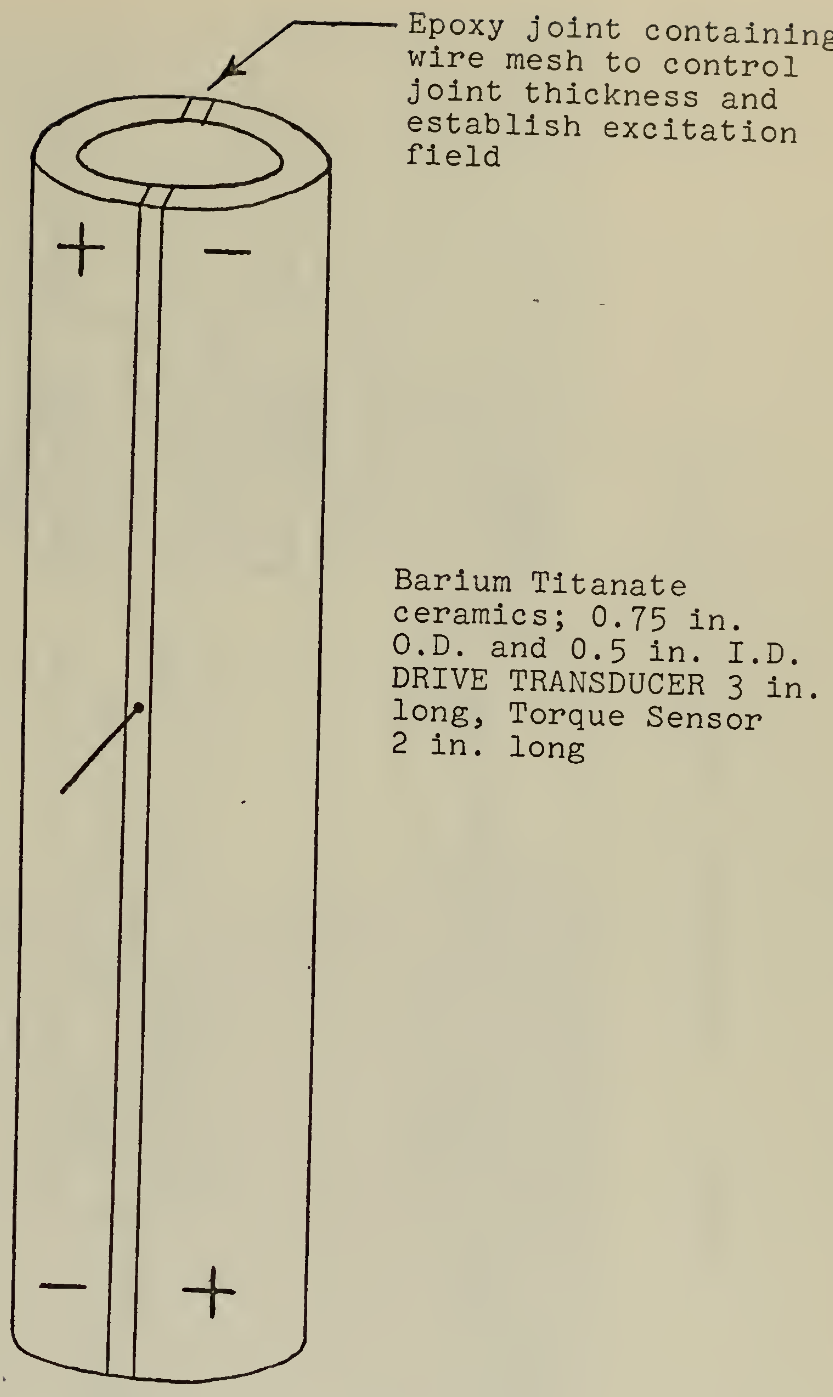


Figure 4. Sketch of Torsional Transducer

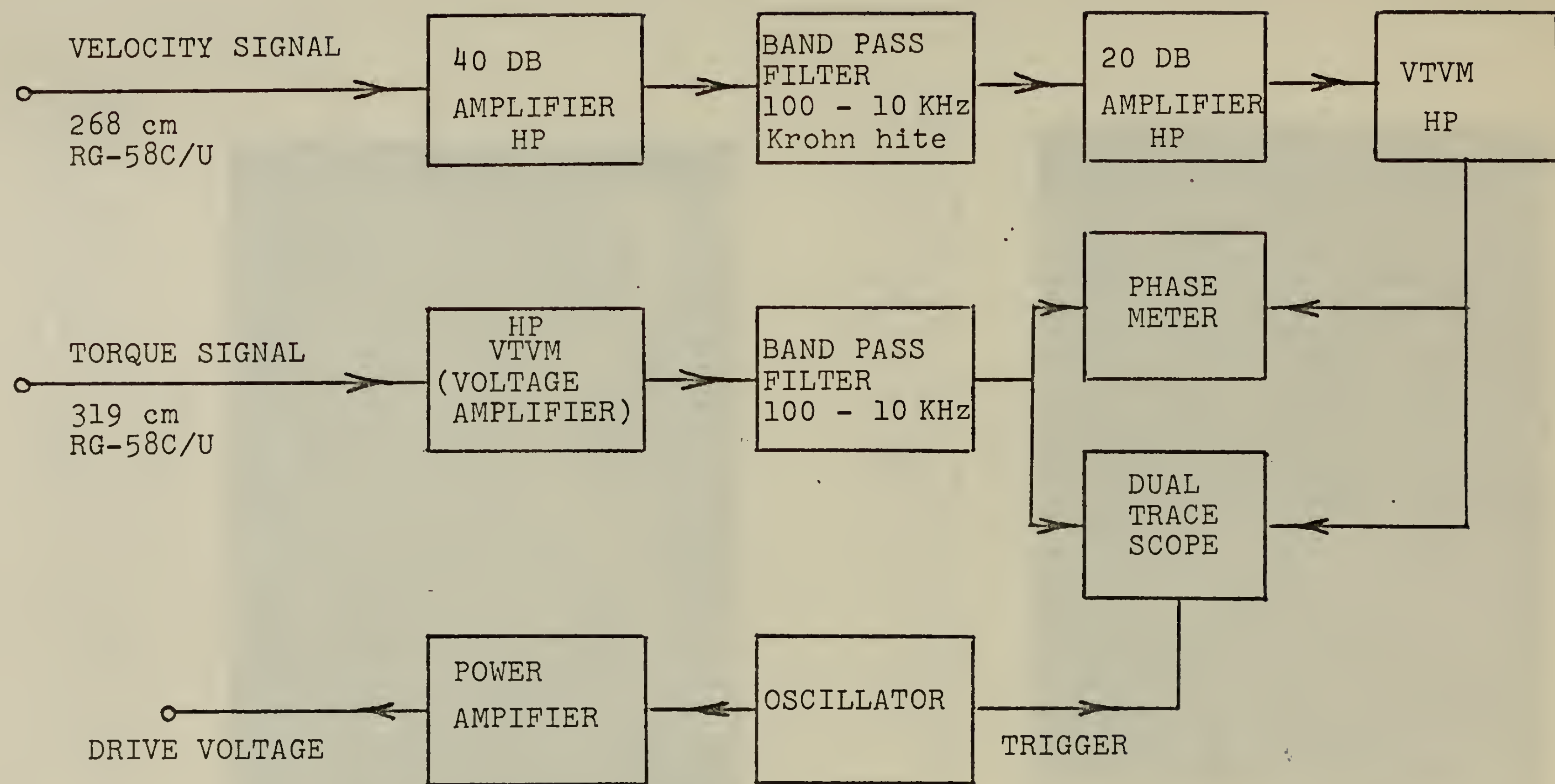


Figure 5. Experimental Set-Up for Testing

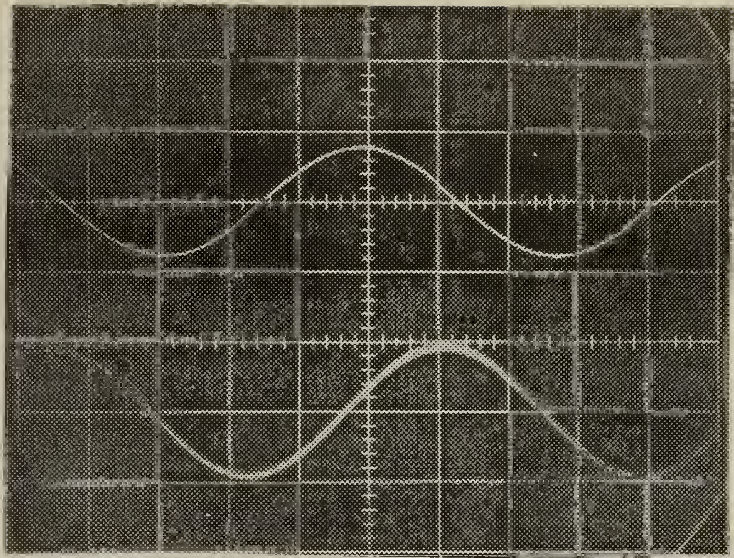


Figure 6A

Observed Torque and Velocity Signals in Air (torque signal equals 7.90 v RMS; velocity signal equals 0.44 mv RMS; phase angle equals +75.1°)

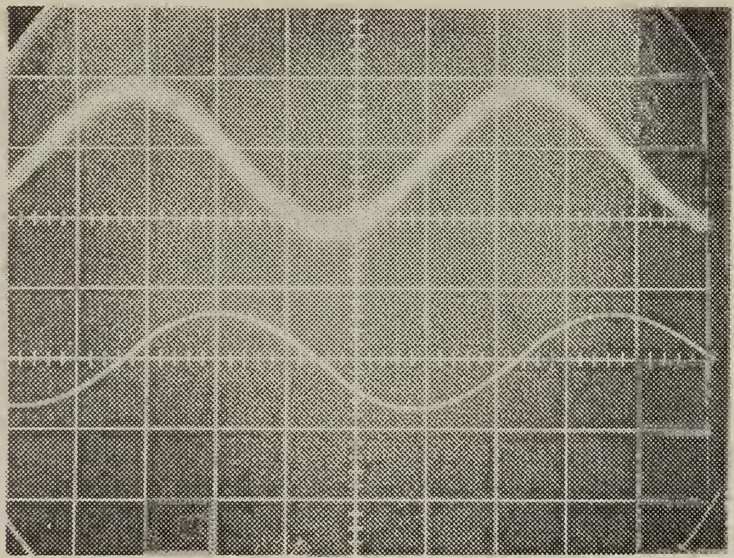


Figure 6B

Observed Torque and Velocity Signals in Mud (torque signal equals 0.324 v RMS; Velocity signal equals 0.0165 mv RMS; phase angle equals 89°)

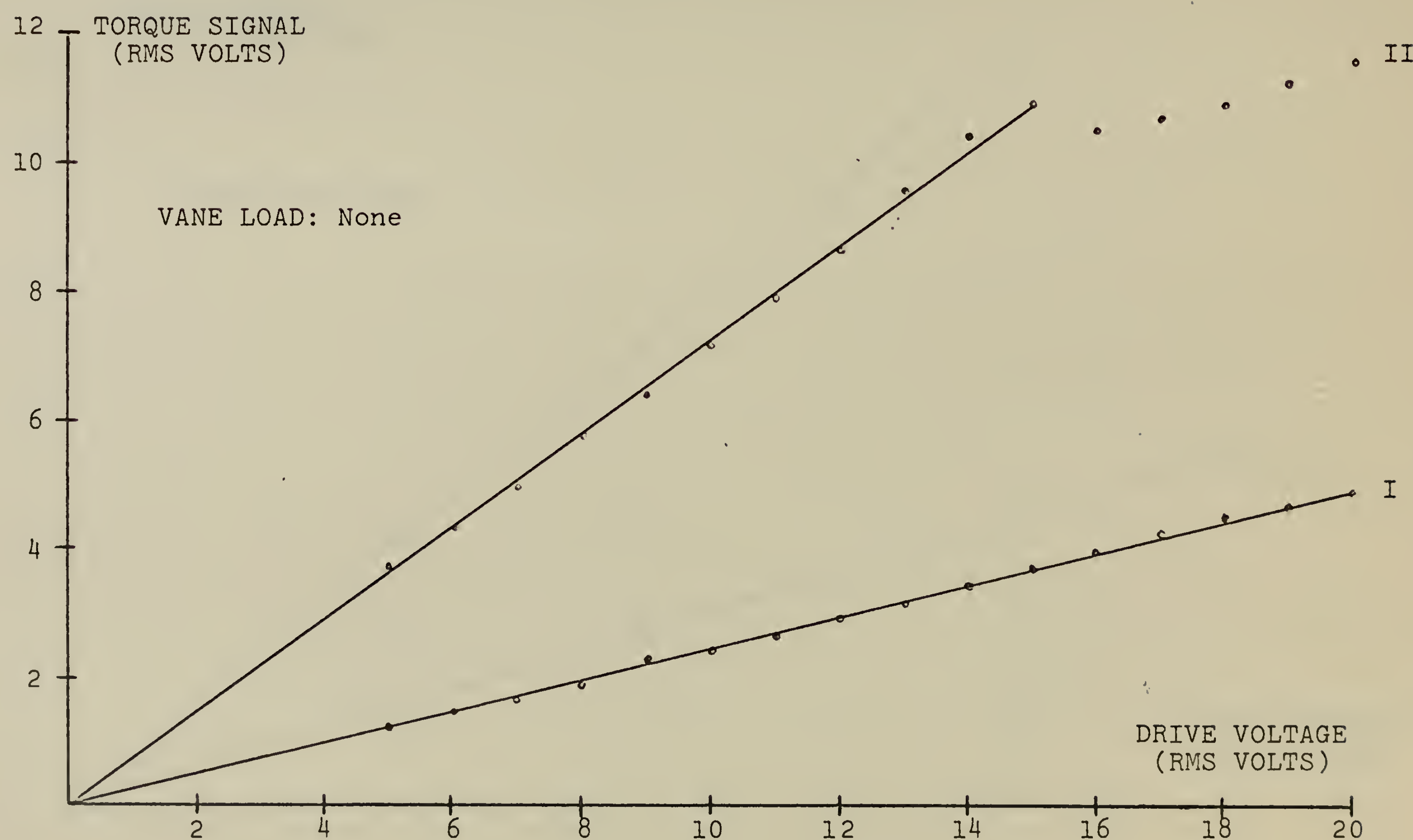


Figure 7. Torque Signal vs. Drive Voltage, No Load

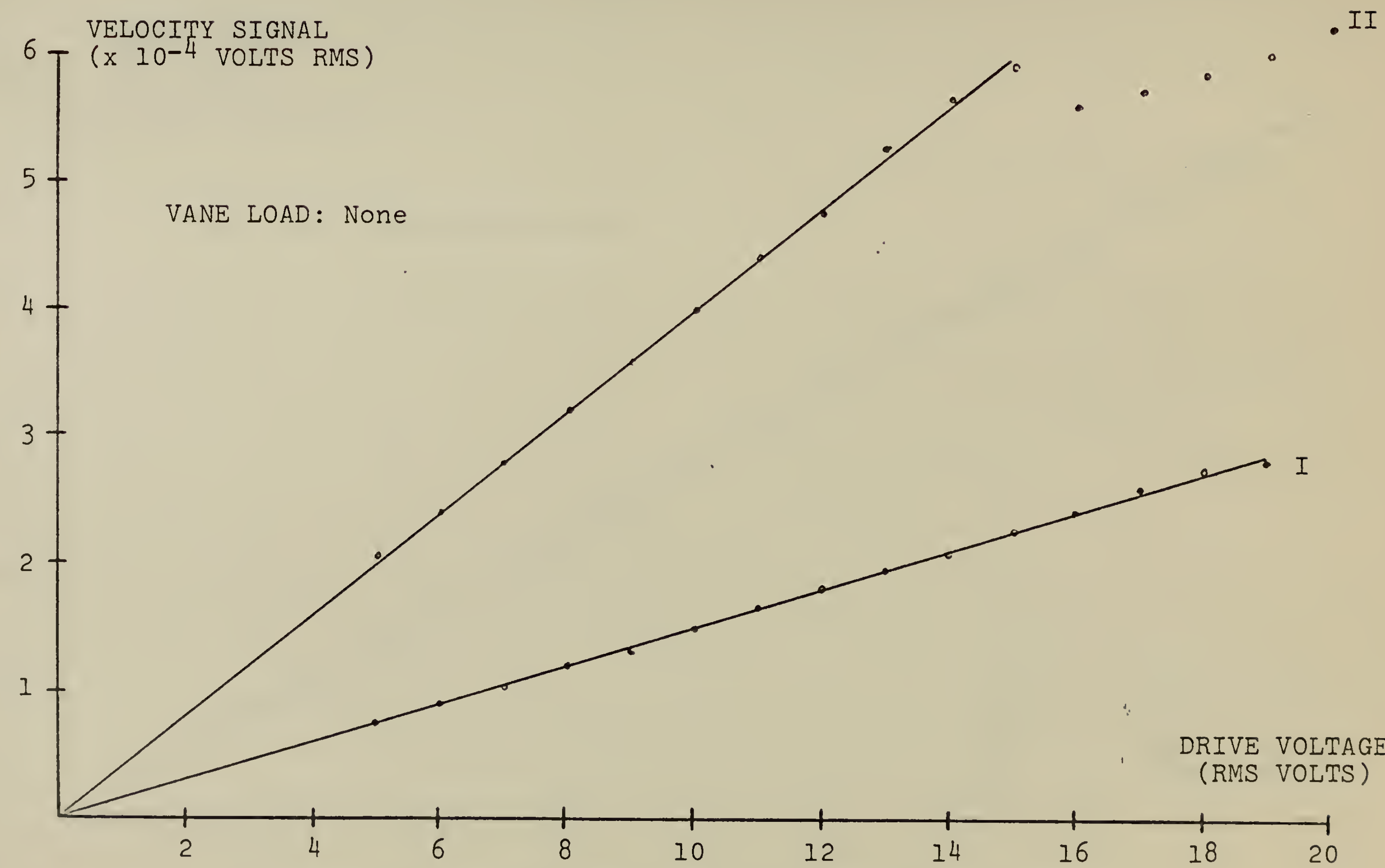


Figure 8. Velocity Signal vs. Drive Voltage, No Load

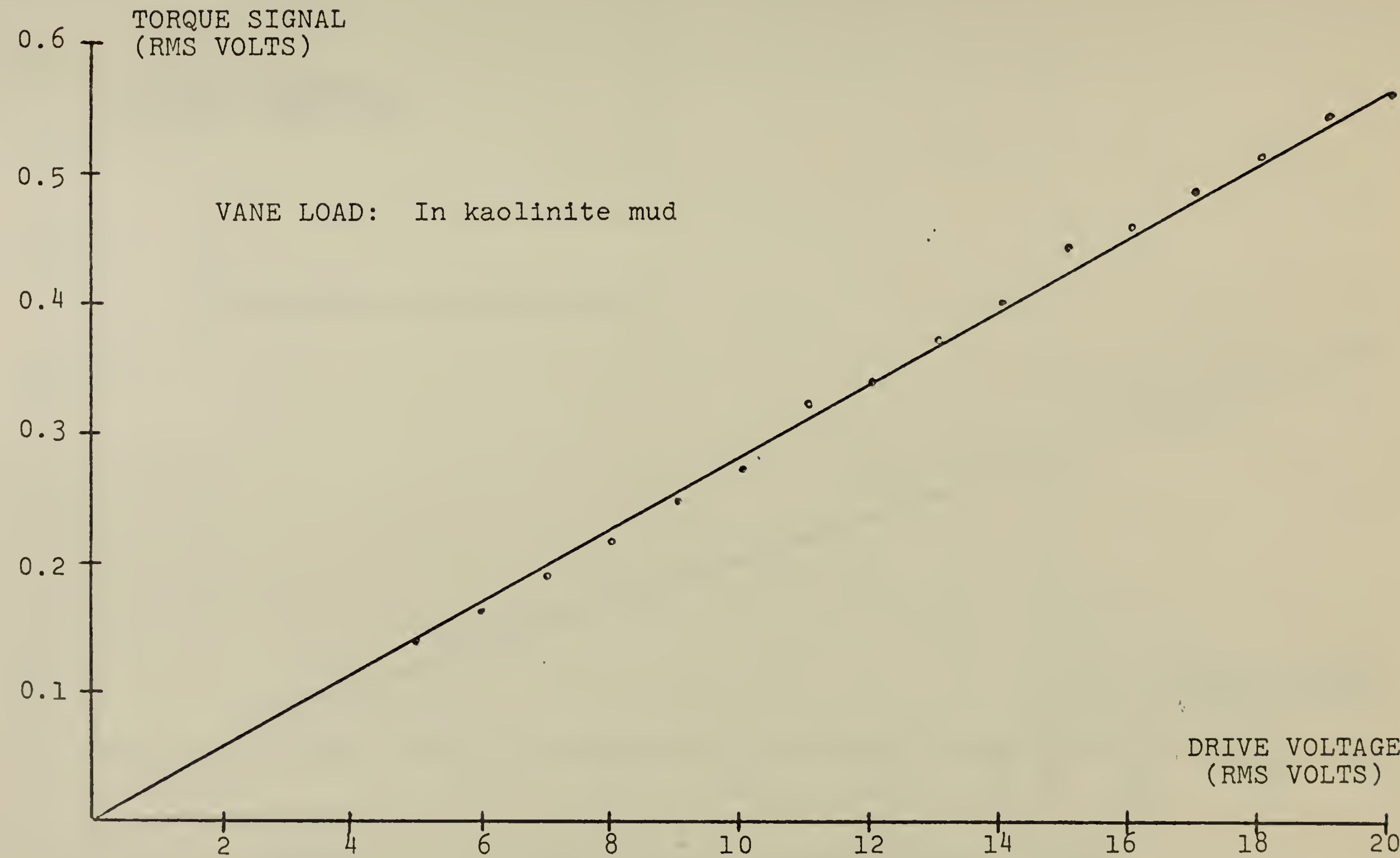


Figure 9. Torque Signal vs. Drive Voltage, in Mud

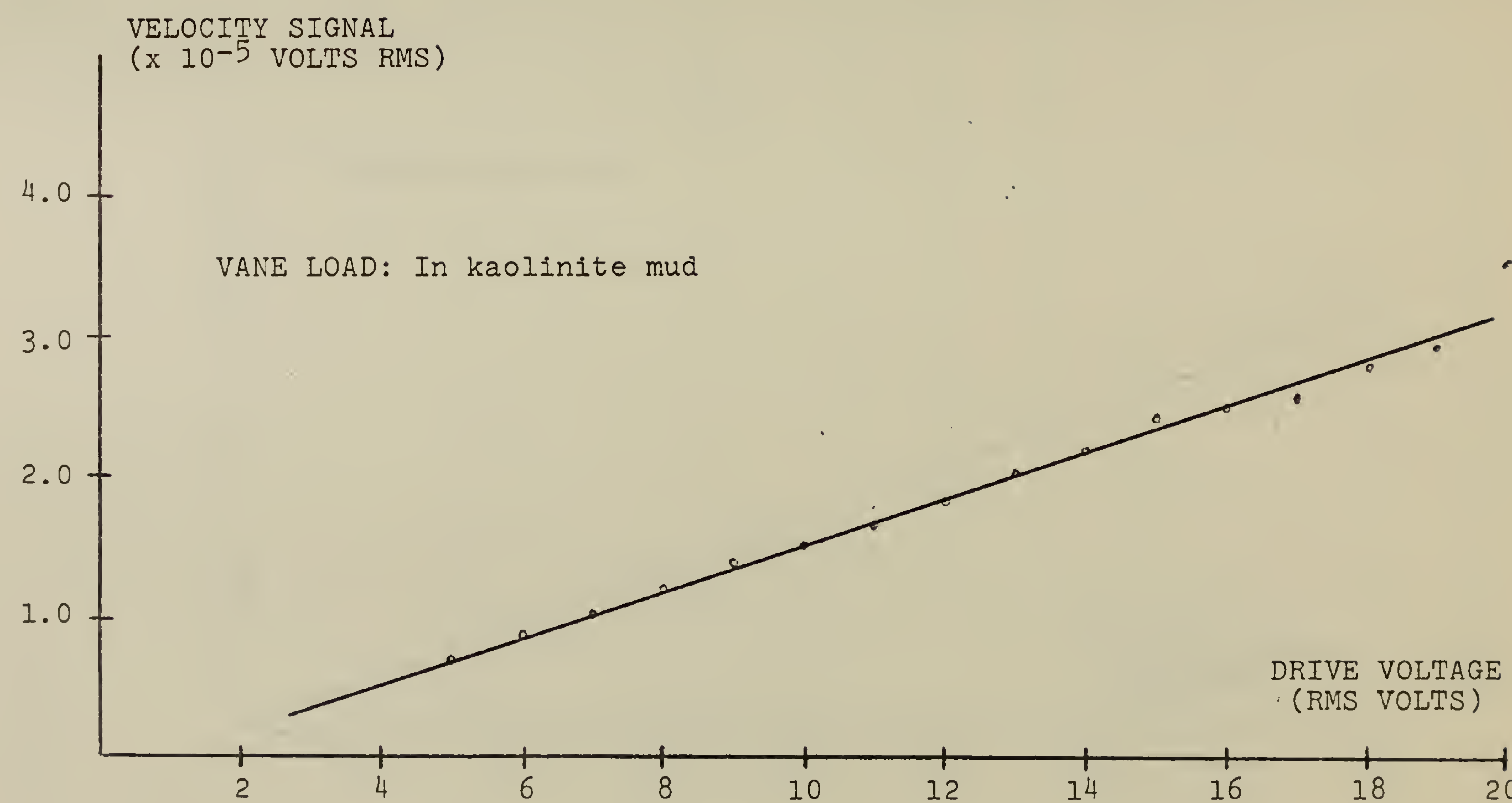


Figure 10. Velocity Signal vs Drive Voltage in Mud

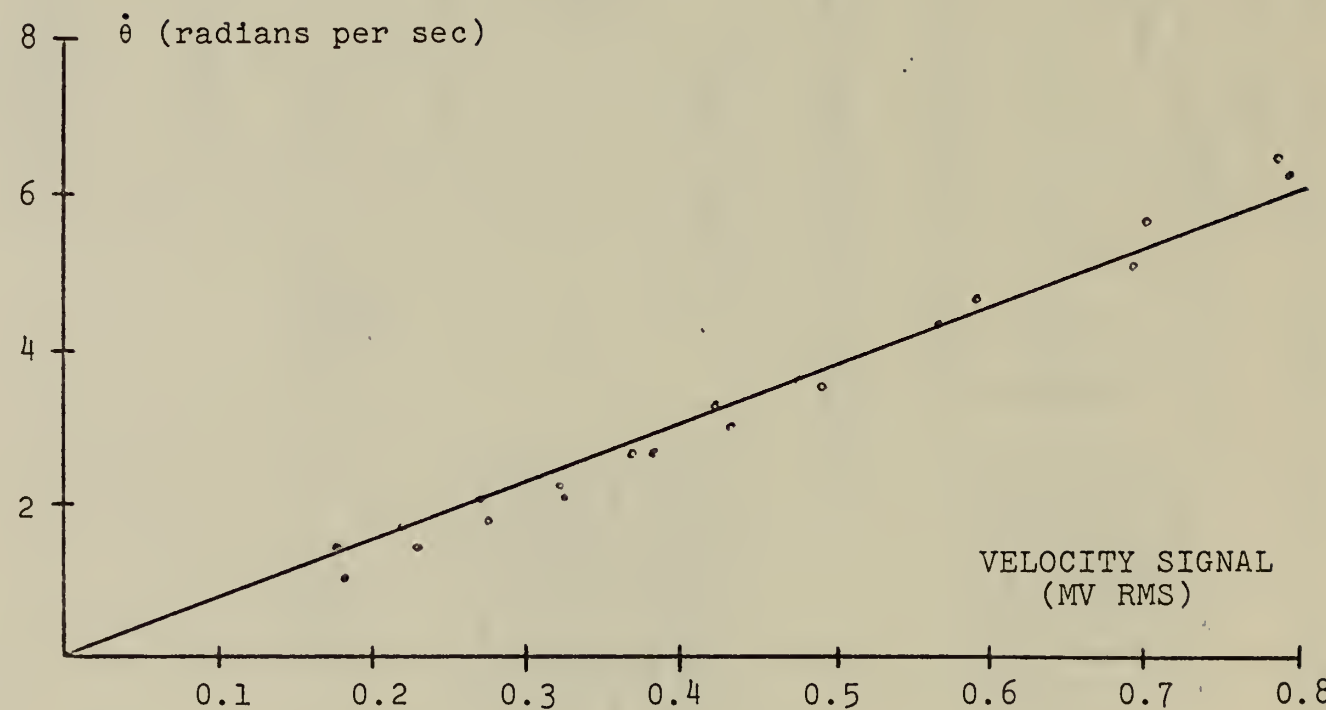
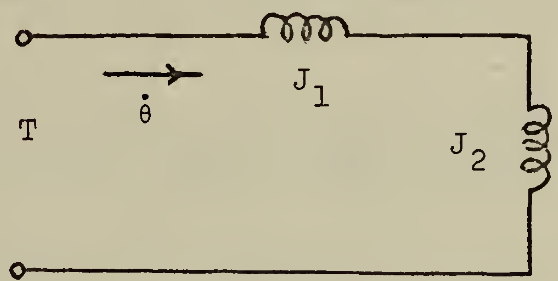


Figure 11. Angular Velocity as a Function of Velocity Sensor Output

APPENDIX I

TORQUE SENSOR CALIBRATION ANALYSIS

An analogous electrical circuit for the mechanical impedance of the head is



where: J_1 is moment of inertia of probe head and J_2 is moment of inertia of added disk. It is assumed that energy dissipation is negligible.

For the no load condition, $J_2 = 0$ and:

$$T = J_1\ddot{\theta}; \qquad K_T V_T = i\omega K_\theta V_\theta J_1$$

then $J_1 = \dfrac{K_T V_T}{i\omega K_\theta V_\theta}$

where K_T and K_θ are calibration constants for the torque and velocity sensors respectively, ω is the angular frequency and V_T and V_θ are the respective sensor output voltages.

Under load conditions,

$$T' = (J_1+J_2)\ddot{\theta}'; \quad K_T V_T' = \left(\frac{K_T V_T}{i\omega K_\theta V_\theta} + J_2 \right) i\omega' K_\theta V_\theta'$$

Solving for K_T yields:

$$K_T = \frac{\omega' K_\theta V_\theta' J_2}{V_T' - \dfrac{\omega'}{\omega}\dfrac{V_\theta'}{V_\theta} V_T}$$

Moments of inertia of Disks are:

Disk 1: 19.15 gm-cm^2 Disk 3: 130.9 gm-cm^2

Disk 2: 61.10 gm-cm^2 Disk 4: 249.0 gm-cm^2

As a sample calculation consider the data for disk 1

$$K_T = \frac{2\pi \times 987 \times 7.46 \times 9.62 \times 10^{-1} \times 19.1}{14.65 - \frac{897}{906} \times \frac{9.62}{10.1} \times 14.2} = \frac{7.72 \times 10^5}{1.27}$$

$$= 6.08 \times 10^5 \text{ dyne-cm-volt}^{-1}$$

CALIBRATION TEST DATA

Load Condition	Torque Sensor Output RMS Volts	Velocity Sensor Output RMS Volts	Frequency of Oscillation Hz
No Load	14.2	10.1×10^{-4}	906
Disk 1.	14.65	9.62×10^{-4}	897
Disk 2.	15.35	8.88×10^{-4}	875
Disk 3.	16.90	8.25×10^{-4}	847
Disk 4.	16.90	6.70×10^{-4}	802

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13. ABSTRACT

The design considerations and the construction details of a torsionally oscillating vaned probe viscoelastometer to be used for measurement of the shear modulus of soft sediments are presented. Its operation involves measuring the reaction to the radiation of shear waves into the sediment from a vaned cylindrical probe which executes simple harmonic torsional oscillations. The radiation reaction should be sensitive to the viscoelastic parameters of the sediment. Torque and angular velocity sensors at the probe head provide a means of measuring the mechanical impedance. Sensor calibration procedures and their results are described. The instrument operates at frequencies in the neighborhood of 900 and 2700 Hz.

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